MATH 170 – CHAPTER 5   Name: .

5.1 Proving Trig Identities

Need To Know

- Recall basic identities
- Recall strategies for proving
- Practice proofs
- Quiz on identities coming soon

Basics on Identities and Proof

Strategy for Proving Identities

1) _____

2) Transform the right side into an expression (A).
   Next transform the left side into the same (A).
   _____

Hints and Tools

a) _____
b) Look for basic Trigonometric Identities that you can substitute into the expression.
c) Look for algebra that you can do to simplify (e.g. add fractions, multiply, factor)
d) _____
   what must the expression finally turn into.

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Ration Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Prove

Prove: $(\sin^2 x)(\cot^2 x + 1) = 1$

Prove

Prove: $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x$

Prove

Prove: $\csc^4 x-\cot^4 x=\dfrac{1+\cos^2 x}{\sin^2 x}$

Prove

Prove: $\dfrac{1-\cos^3 A}{1-\cos A}=\cos^2 A+\cos A+1$

end

5.2 Sum & Differences Identities

<u>Need To Know</u>

- Recall Even and Odd identities
- Recall Cofunction identities
- Develop proof for sum & diff. identities
- Applications

Identities and Counter Example

sin (-θ) = _____

cos (-θ) = _____

sin (θ) = cos (_____)

cos (θ) = sin (_____)

Guess:

cos(A + B) = _____

(check for counter example)

Construct Angle Sum Identity

Draw a unit circle, ∠A, ∠A+B, ∠-B

Prove the chord distances are congruent.
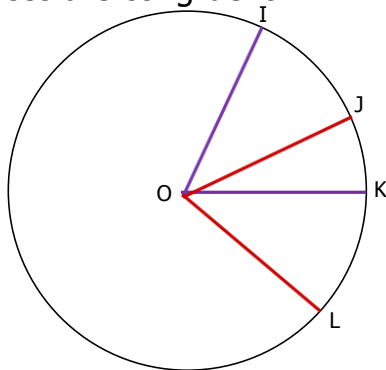
<u>Given</u>:

OI   =   OJ

OK   =   OL

∠IOK= ∠ JOL

<u>so</u>

Construct Angle Sum Identity

So the   distance of IK   =   distance of JL

$[\cos(A + B) - 1]^2 + [\sin(A + B) - 0]^2 = (\cos A - \cos B)^2 + (\sin A + \sin B)^2$

$\cos^2(A + B) - 2\cos(A + B) + 1 \quad = \quad \cos^2 A - 2\cos A\cos B + \cos B^2$

$\sin^2(A + B) \qquad\qquad\qquad\qquad + \sin^2 A + 2\sin A \sin B + \sin B^2$

Angle Sum and Diff Identities

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Application

Find the exact value:

$\sin 75° =$

$\cos \pi/12 =$

Application

Sketch the graph y on [0, 2π]

y = sin x cos 2x + cos x sin 2x

Application

Prove: $\dfrac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$

end

5.3 Double Angle Identities

Need To Know

- Recall angle sum identities
- Develop double angle identities
- Apply

Double Angle of Sine

Recall sin(A + B) =

Double Angle of Cosine

Recall cos(A + B) =

Angle Sum and Diff Identities

_____

_____

$$= \text{_____}$$
$$= \text{_____}$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

Application

If $\cos A = \dfrac{2}{\sqrt{7}}$ with A in QIV, find $\sin 2A$ and $\sec 2A$

Application

Simplify:

$\cos^2 15° - \sin^2 15° =$

$\sin \dfrac{\pi}{8} \cos \dfrac{\pi}{8} =$

Application

Simplify:

$\cos^2 15° - \sin^2 15° =$

$\sin \dfrac{\pi}{8} \cos \dfrac{\pi}{8} =$

Proof Practice

Prove: cos3θ = 4cos³θ − 3cosθ

Proof Practice

Prove: $\csc\theta - 2\sin\theta = \dfrac{\cos 2\theta}{\sin\theta}$

end

5.4 Half Angle Identities

Need To Know

- Recall Double Angle Identities
- Develop Half Angle Identities
- Apply
  - Exact values
  - Graphs
  - Proof

Half Angle Identity for Cosine

Recall: $\cos 2x = 2\cos^2 x - 1$

Solve for cos x
and set 2x = A

Half Angle Identities

Choose the + or − based upon which
quadrant that the angle A/2 is in.

Practice

If $\sin B = -12/13$ with $180° \leq B \leq 270°$,
find sine, cosine and tangent of B/2.

Apply to Graphing

Sketch a graph of $y = 6\cos^2\dfrac{x}{2}$

Exact values

Evaluate sin 105°

Apply to Proofs

Prove: $\csc^2\dfrac{A}{2} = \dfrac{2\sec A}{\sec A - 1}$

end

5.5 Additional Identities

<u>Need To Know</u>

- More Trig Id with Inverse Trig functions
- Product and Addition formulas
- Apply to proof

Trig Functions & Inverse Functions

Find the exact value of $\quad \sin\left(\tan^{-1}\dfrac{1}{2}-\sin^{-1}\dfrac{1}{2}\right)$

Trig Functions & Inverse Functions

Express in terms of x only: $\sin(2\cos^{-1}x)$

Product and Sum Formulas

Use the identities in the book to rewrite & simplify

cos 5x − cos3x

$$\sin\frac{\pi}{12}+\sin\frac{7\pi}{12}$$

Apply to Proof

Prove: $-\tan 4x = \dfrac{\cos 3x - \cos 5x}{\sin 3x - \sin 5x}$

end